Spin current and spin accumulation near a Josephson junction between a singlet and triplet superconductor

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We consider a Josephson junction with an arbitrary transmission coefficient $\mathcal{D}$ between a singlet and a triplet superconductor with the latter order parameter characterized by a d vector of the form $(k_x\hat{y}-k_y\hat{x})$. Various quantities such as the tunneling current, spin accumulation, and spin current are calculated via the quasiclassical Green's functions. We also present a symmetry argument on the existence of these quantities and their dependencies on the phase difference across the junction. A physical picture is also given in terms of the Andreev states near the junction.

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I. INTRODUCTION

Recently, there has been much interest in manipulating the spin degree of freedom of electrons in condensed-matter systems. Phenomena such as spin current, spin-Hall effect, spin accumulation, and magnetoelectric effects have received a lot of attention.[1] These phenomena have been discussed in a variety of systems, including metals, semiconductors, and even insulators. In this paper, we discuss the spin current and spin accumulation near a Josephson junction. We shall, in particular, consider a junction between an s-wave superconductor and a pure-triplet superconductor with the latter in the state where the "d vector," specifying the spin structure of the Cooper pairs, is given by $\hat{d}=k_x\hat{y}-k_y\hat{x}$.

We are interested in this $k_x\hat{y}-k_y\hat{x}$ state for a number of reasons. This state corresponds to the one-dimensional representation A_{2u} in a crystal with tetragonal D_{4h} symmetry[2] and hence is one of the simplest example of a triplet state. This state is also believed to be a limiting case for the order parameter of the noncentrosymmetric superconductor[3] $CePt_xPd_{3-x}Si$. There, due to the absence of inversion symmetry in the normal state, the order parameter is believed to be a mixture of the s-wave and the p-wave A_{2u} order parameters. [The state $k_x\hat{y}-k_y\hat{x}$ is the limiting case (perhaps for small Pd concentration $3-x$) where the s-wave admixture is small.] Similar mixing of superconducting order parameter of different parity is also expected in compounds such as $CeRhSi_3$ (Ref. 4) and in superconductivity found at oxide interface.[5] Spin current generated near the surface of this superconducting state with vacuum have been discussed recently by two groups,[6,7] with and without the mixing of the s-wave order parameter due to the absence of inversion symmetry.[8]

We generalized these considerations to the case where this superconductor is in contact with an s-wave superconductor in the form of a Josephson junction with arbitrary transmission coefficient $\mathcal{D}$, but for simplicity we shall not include any broken-inversion-symmetry effects in the normal state; hence the bulk superconductors are assumed to be pure singlet and pure triplet, respectively. Clearly, in the infinitely high barrier limit, our results would just be a special case of Refs. 6 and 7.

For general transmission, however, one expects a proximity effect so that near the interface, the system acquires properties of a superconductor with mixed singlet and triplet order parameters similar to the case which arises in noncentrosymmetric superconductors,[3–5] even though our bulk superconductors are each purely singlet and triplet. Effects that are normally not allowed can now appear due to the lowering of symmetries, similar to the electro-magneto effects discussed recently for bulk noncentrosymmetric superconductors.[9] There, in particular, a supercurrent can generate a spin polarization in a perpendicular direction. Here, we shall investigate how the spin current (and the spin accumulation) depends on (and hence can be manipulated by) the phase difference between these two superconductors.

Our investigation is interesting in another point of view. The state $k_x\hat{y}-k_y\hat{x}$ has two counterpropagating edge states of opposite spins near a surface (see below), in direct analogy with the quantum spin-Hall state often discussed in the current literature.[10,11] The investigation here is analogous to considering an interface between an ordinary "insulator" (our s-wave superconductor) and a "quantum spin-Hall insulator" (our $k_x\hat{y}-k_y\hat{x}$ superconductor). Discussions on this and other related triplet superconductors from this point of view can also be found in Refs. 12 and 13.

A recent paper[14] also studies the spin accumulation near a Josephson junction between a pure-singlet and pure-triplet superconductor. In that paper, only the very special p-wave state, where $\hat{d}$ is independent of the momentum direction $\hat{k}$, was considered. Spin accumulation was shown to exist near the junction with the spin direction along $\hat{d}$. The authors suggested the detection of this spin accumulation as a method of identifying triplet superconductors. However, the constant $\hat{d}$ vector is a very special case. A general triplet superconductor is expected to have $\hat{k}$-dependent d vectors.[2] For these more general cases, it is then unclear if spin accumulation would exist and in which direction the net spin lies. We would like to provide a general consideration using this $(k_x\hat{y}-k_y\hat{x})$ state as an illustrative example.

Our paper is organized as follows. We begin with a symmetry argument in Sec. II. We then present our calculations with the quasiclassical method in Sec. III. The subsections provides our results, first for the special cases of perfect and small transmissions, then the more general case with arbitrary $\mathcal{D}$. We summarize in Sec. IV. We employ a generaliza-

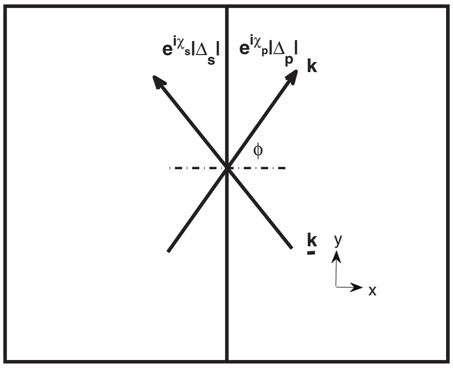

FIG. 1. A schematic view of the singlet-triplet junction. The triplet superconductor, with an order parameter of magnitude $|\Delta_p|$ and phase χ_p, occupies the right $(x>0)$ while the singlet one, whose respective values denoted by $|\Delta_s|$ and χ_s, occupies the left $(x<0)$. The quasiclassical path is denoted by the direction of quasiparticle momentum $\hat{k}$. The angle ϕ is defined with respect to the x axis. Incoming and outgoing paths labeled by $\underline{\hat{k}}$ and $\hat{k}$, respectively, are used for interface with nonperfect transmission.

tion of the "exploding and decaying trick," which we explain in the Appendix.

II. JUNCTION GEOMETRY AND SYMMETRY CONSIDERATIONS

We shall then consider a Josephson junction between an s-wave superconductor and a purely triplet superconductor with $\hat{d}=k_x\hat{y}-k_y\hat{x}$. For simplicity we shall consider the two-dimensional case or, equivalently, the three-dimensional case but dispersionless in k_z. A schematic view of the junction is shown in Fig. 1. We shall show that symmetry argument forbids the existence of certain quantities and, in the case where a quantity is allowed, its dependence on the phase difference is constrained. We search for symmetry operations under which the junction would map back to itself. Caution has to be taken to account for possible changes in the phase of the order parameters under these operations. These considerations are along the same line, as those applied earlier by one of us,[15,16] to the Josephson current across a junction.

The s-wave (triplet) superconductor occupies $x<(>)0$. The order parameter $\underline{\Delta}$ is a 2×2 matrix in spin space. We have, for $x<0$, $\underline{\Delta}=\Delta_s(i\sigma_y)$ whereas for $x>0$, $\underline{\Delta}(\hat{k})=\Delta_p i(\vec{d}(\hat{k})\cdot\vec{\sigma})\sigma_y$, where $\vec{d}(\hat{k})=\hat{k}_x\hat{y}-\hat{k}_y\hat{x}$ specifies the triplet structure of the pairs. We shall, for simplicity, ignore the anisotropy of the magnitude of the superconducting gaps. In this case, Δ_s and Δ_p are independent of $\hat{k}$.

First we consider the time-reversal transformation Θ under which the supercurrent and spin accumulation are odd while the spin current is even. The annihilation operators transform as $\Theta a_{\vec{k},\uparrow}\Theta^{-1}=a_{-\vec{k},\downarrow}$ and $\Theta a_{\vec{k},\downarrow}\Theta^{-1}=-a_{-\vec{k},\uparrow}$. Using the fact that $\underline{\Delta}$ transforms as the corresponding anomalous average, simple algebra then shows that $\Delta_s\to\Delta_s^*$ and $\Delta_p\to\Delta_p^*$ with $\hat{d}$ unchanged (using $\hat{d}$ is real). Hence the phase difference χ changes sign. It follows that the supercurrent $J_j(\chi)=-J_j(-\chi)$, spin accumulation $S^i(\chi)=-S^i(-\chi)$, and spin current $J_j^i(\chi)=J_j^i(-\chi)$ for polarization and flow along i and j, respectively.

Under a reflection in the x-z plane, the order parameter $\hat{d}=k_x\hat{y}-k_y\hat{x}$ transforms according to $(k_x,k_y,k_z)\to(k_x,-k_y,k_z)$ and $(\hat{x},\hat{y},\hat{z})\to(-\hat{x},\hat{y},-\hat{z})$, respectively. Hence both superconductors are invariant and the phase difference χ is also unchanged. The only nonvanishing currents, spins, and spin currents allowed are thus $J_{x,z}$, S^y, $J_{x,z}^y$, and $J_y^{x,z}$. Since the dispersion in z is not considered, J_z and J_z^y will not be mentioned hereafter. We can also consider a reflection in the x-y plane under which $(k_x,k_y,k_z)\to(k_x,k_y,-k_z)$ and $(\hat{x},\hat{y},\hat{z})\to(-\hat{x},-\hat{y},\hat{z})$. The resulting order parameter $\Delta_s\to\Delta_s$ but $\Delta_p\to\Delta_p e^{i\pi}$, hence the phase difference $\chi\to\chi+\pi$. We then have

$$J_x(\chi)=J_x(\chi+\pi),$$

$$S^y(\chi)=-S^y(\chi+\pi),$$

$$J_x^y(\chi)=-J_x^y(\chi+\pi),$$

$$J_y^x(\chi)=-J_y^x(\chi+\pi),$$

$$J_y^z(\chi)=J_y^z(\chi+\pi). \tag{1}$$

Other symmetry operations (such as π rotation about $\hat{x}$) just produce relations that can be found by combinations of those listed above. We note, in particular, that the spin accumulation lies entirely along the y direction. In the limit of zero transmission, all quantities are independent of χ. In this case, all spin accumulations must vanish and the only finite spin current is J_y^z. These results hold even when more general components of the A_{2u} order parameter [e.g., $k_xk_y(k_x\hat{x}-k_y\hat{y})$ in Ref. 2] are included. As we shall see later, only J_x, S^y, and J_y^z are found to be finite in our calculations.

III. QUASICLASSICAL GREEN'S FUNCTION

We now present our calculations and the quasiclassical method. At positions other than the interface, the quasiclassical energy-integrated Green's function $\hat{g}$, a function of momentum direction $\hat{k}$, Matsubara frequency ϵ_n, and position $\vec{r}$, obeys[17]

$$[i\epsilon_n\tau_3-\hat{\Delta},\hat{g}]+i\vec{v}_f(\hat{k})\cdot\vec{\nabla}\hat{g}=0, \tag{2}$$

with the normalization condition

$$\hat{g}^2=-\pi^2\hat{1}. \tag{3}$$

Here $\vec{v}_f(\hat{k})$ is the Fermi velocity. The boundary condition at $x=0$ will be stated below. $\hat{\Delta}(\hat{k})$ specifies the off-diagonal pairing field. $\hat{\Delta}=\begin{pmatrix}0&\underline{\Delta}\\-\underline{\Delta}^\dagger&0\end{pmatrix}$ where $\underline{\Delta}$ is the 2×2 order-parameter matrix in spin space. With $\tau_+\equiv\begin{pmatrix}0&1\\0&0\end{pmatrix}$ and $\tau_-\equiv\begin{pmatrix}0&0\\1&0\end{pmatrix}$ in particle-hole space, we then have $\hat{\Delta}=\Delta_s(i\sigma_y)\tau_+ + \Delta_s^*(i\sigma_y)\tau_-$ for $x<0$ and $\hat{\Delta}=\Delta_p i[\vec{d}(\hat{k})\cdot\vec{\sigma}]\sigma_y\tau_+ + \Delta_p^* i\sigma_y[\vec{d}(\hat{k})\cdot\vec{\sigma}]\tau_-$ for $x>0$. In order to have tractable analytic solutions for $\hat{g}$, we shall also ignore the self-consistent gap equation and hence the spatial dependence of Δ_s and Δ_p. In order to explain the physics without unnecessary complications, we shall also assume for simplicity that each of the superconductor has a single Fermi surface with the Fermi velocity magnitudes of the two superconductors identical and independent of $\hat{k}$. Moreover, we shall assume that the barrier is smooth so that momentum parallel to the junction is a good quantum number.

A. Perfect transmission

In this case, the boundary condition at $x=0$ is simply that $\hat{g}$ is continuous. $\hat{g}(\hat{k},\epsilon_n,0)$ is given by (see the Appendix)

$$\hat{g}(\hat{k},\epsilon_n,0) = -i\pi\{\hat{a},\hat{b}\}^{-1}[\hat{a},\hat{b}], \tag{4}$$

where [,] and {,} denote commutators and anticommutators and $\hat{a}$ and $\hat{b}$ the appropriate exponentially decaying and increasing solutions along the quasiclassical path.

For $k_x>0$, we need $\hat{a}=\hat{a}_p$ and $\hat{b}=\hat{b}_s$ in Eq. (4). We find

$$\hat{g}(\hat{k},\epsilon_n,0) = c_3(\hat{k},\epsilon_n)\tau_3 + c_3'(\hat{k},\epsilon_n)\begin{pmatrix} \hat{d}(\hat{k})\cdot\vec{\sigma} & 0 \\ 0 & -\sigma_y\hat{d}(\hat{k})\cdot\vec{\sigma}\sigma_y \end{pmatrix} + (\text{o.d.}), \tag{5}$$

where (o.d.) denotes off-diagonal terms in particle-hole space that we would not need,

$$c_3(\hat{k},\epsilon_n) = -\pi\frac{|\Delta_p|^2|\Delta_s|^2\sin\chi\cos\chi + i\epsilon_n(\alpha_s\alpha_p+\epsilon_n^2)(\alpha_p+\alpha_s)}{(\alpha_s\alpha_p+\epsilon_n^2)^2-|\Delta_p|^2|\Delta_s|^2\cos^2\chi}, \tag{6}$$

$$c_3'(\hat{k},\epsilon_n) = \pi|\Delta_p||\Delta_s|\frac{(\alpha_s\alpha_p+\epsilon_n^2)\sin\chi + i\epsilon_n(\alpha_p+\alpha_s)\cos\chi}{(\alpha_s\alpha_p+\epsilon_n^2)^2-|\Delta_p|^2|\Delta_s|^2\cos^2\chi}, \tag{7}$$

where $\chi\equiv\chi_p-\chi_s$ is the phase difference. $\alpha_s\equiv(\epsilon_n^2+|\Delta_s|^2)^{1/2}$ and $\alpha_p\equiv(\epsilon_n^2+|\Delta_p|^2)^{1/2}$. The result for c_3 was also given in Ref. 15. For $k_x<0$, we need $\hat{a}=\hat{b}_p$ and $\hat{b}=\hat{a}_s$ in Eq. (4). Alternatively, we can also use the symmetry [Eq. (C.10c) in Ref. 17] $\hat{g}(-\hat{k},-\epsilon_n)=\tau_2\hat{g}^{tr}(\hat{k},\epsilon_n)\tau_2$, where *tr* denotes the transpose. $\hat{g}(\hat{k},\epsilon_n)$ is still of the form in Eq. (5), with $c_3(-\hat{k},-\epsilon_n)=-c_3(\hat{k},\epsilon_n)$ and $c_3'(-\hat{k},-\epsilon_n)=-c_3'(\hat{k},\epsilon_n)$. Note that we have defined the c_3 and c_3' coefficients with $\hat{k}$-dependent $\hat{d}$ vector in Eq. (5) and $\hat{d}(-\hat{k})=-\hat{d}(\hat{k})$.

The number current density along x can in general be expressed as

$$J_x = \frac{1}{2}N_f v_f\int\frac{d\phi}{2\pi}(\cos\phi)T\sum_n \text{Tr}[\tau_3\hat{g}(\hat{k},\epsilon_n)], \tag{8}$$

where ϕ is the angle of $\hat{k}$ with respect to $\hat{x}$ and N_f is the density of states per unit area for a single spin species. The symbol Tr represents taking a full trace in both the spin and particle-hole spaces. Only the c_3 component in Eq. (6) contributes to J_x. The spin density in the i direction at $x=0$ can be expressed as[18]

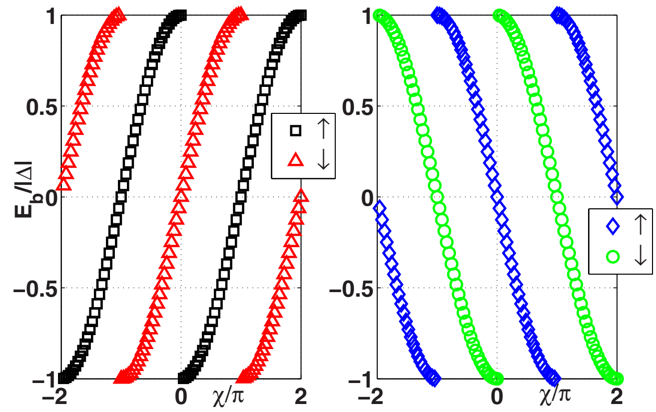

FIG. 2. (Color online) The interface bound states associated with the $k_x>0$ and $k_x<0$ paths in the perfect transmission case. To facilitate the discussions for spin accumulation and spin current, ↑ spin means parallel to $\hat{d}(\hat{k})$ associated with the right-moving path, i.e., $k_x>0$ but antiparallel to $\hat{d}(\hat{k})$ if $k_x<0$.

$$S^i = \frac{\hbar}{4}N_f\int\frac{d\phi}{2\pi}T\sum_n \text{Tr}[\hat{\sigma}^i\hat{g}(\hat{k},\epsilon_n)]. \tag{9}$$

Here we define the symbols $\hat{\sigma}^i$ by $\hat{\sigma}^x\equiv\sigma_x$, $\hat{\sigma}^y=\sigma_y\tau_3$, and $\hat{\sigma}^z\equiv\sigma_z$. So here only S^y is finite and is associated with c_3' in Eq. (7). The spin current densities, with superscript (subscript) denoting the spin (flow) direction at $x=0$ is[18]

$$J_j^i = \frac{\hbar}{4}N_f v_f\int\frac{d\phi}{2\pi}(\hat{k}_j)T\sum_n \text{Tr}[\tau_3\hat{\sigma}^i\hat{g}(\hat{k},\epsilon_n)]. \tag{10}$$

Note that the three components of $\hat{\sigma}^i\tau_3$ are $\sigma_x\tau_3$, σ_y, and $\sigma_z\tau_3$. It follows that all the spin currents vanish since $\hat{g}$ of Eq. (5) does not contain any $\hat{\sigma}^i\tau_3$ components. Physically, the Andreev equation for each $\hat{k}$ is decoupled from other paths and hence can be block diagonalized using quantization axis along $\hat{d}(\hat{k})$. Along this axis, both the singlet and triplet superconductors consist of only ↑↓ pairs. These Cooper pairs do not have any net spins, and they cannot contribute to any dissipationless spin current. (See also the discussions near the end of Sec. III C.)

Next we present explicit results for the case of equal gaps on both sides, i.e., $|\Delta_s|=|\Delta_p|=|\Delta|$. Here the interface bound states, which correspond to the poles of $\hat{g}$ in Eq. (5), are essential for the quantities in Eqs. (8) and (9). It can be shown that for the right moving path ($k_x>0$), the bound states of spin parallel and antiparallel with $\hat{d}(\hat{k})$ are given by $E_{b,\uparrow}=-|\Delta|\cos(\frac{\chi}{2})\text{sgn}[\sin(\frac{\chi}{2})]$ and $E_{b,\downarrow}=|\Delta|\sin(\frac{\chi}{2})\text{sgn}[\cos(\frac{\chi}{2})]$, respectively. For the left-moving path ($k_x<0$), the bound-state energies are $E_{b,\uparrow}=-|\Delta|\sin(\frac{\chi}{2})\text{sgn}[\cos(\frac{\chi}{2})]$ and $E_{b,\downarrow}=|\Delta|\cos(\frac{\chi}{2})\text{sgn}[\sin(\frac{\chi}{2})]$. Notice that we adopt a common spin-quantization axis for both right- and left-moving paths (caption of Fig. 2) to facilitate the following discussions. The bound-state spectra are plotted as a function of phase difference χ in Fig. 2. It can be seen that for a given path, the two branches of opposite spin projections are identical except

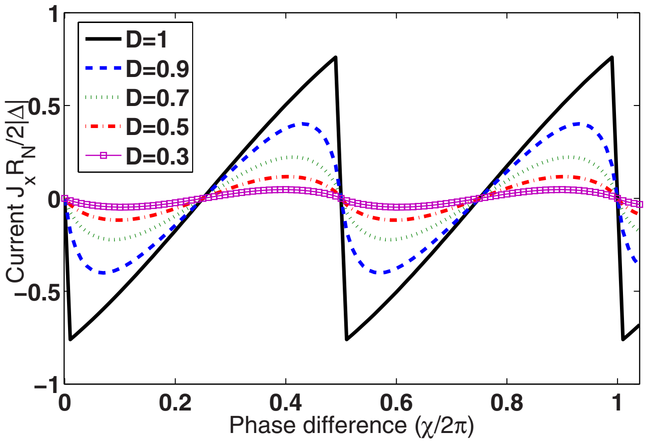

FIG. 3. (Color online) The calculated supercurrent J_x for various transmission coefficients $\mathcal{D}$. The gap on both sides are set to equal to $|\Delta|$ and $T=|\Delta|/100$.

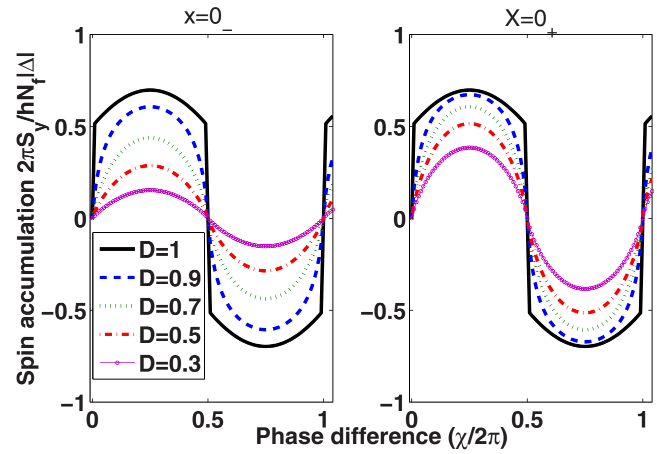

FIG. 4. (Color online) The spin accumulation S^y at $x=0_{\mp}$ on the two sides of the interface for various transmission coefficients $\mathcal{D}$ with $T=|\Delta|/100$.

separated by π, which reflects the invariance of triplet order parameter under $\chi_p \to \chi_p + \pi$ and $\hat{d} \to -\hat{d}$.

The analytical results for J_x is obtained using Eq. (8), which gives

$$J_x = -\frac{2|\Delta|}{e^2 R_N}\frac{\pi}{4}\left[\cos(\chi/2)\tanh\frac{|\Delta|\sin(\chi/2)}{2T} - \sin(\chi/2)\tanh\frac{|\Delta|\cos(\chi/2)}{2T}\right], \quad (11)$$

where R_N denotes the corresponding resistance in the normal state. Equation (11) coincides with the previous results in Ref. 15. J_x is plotted in Fig. 3 and the present case corresponds to the line denoted by $\mathcal{D}=1$. J_x can be understood by summing over contributions $\frac{\partial E_b}{\hbar \partial \chi}$ from occupied bound states. Notice that a current jump occurs whenever χ is a multiple of π. When χ is slightly larger than 0, the state labeled by the square in the left panel and the state labeled by the diamond in the right are occupied. Only the latter bound state with a negative slope contributes to J_x. When χ is slightly less than 0, on the other hand, the state labeled by the triangle in the left panel is occupied and contributes a positive current.

Moreover, the splitting between bound states actually contributes to a finite spin accumulation near the interface along some direction. Consider $0<\chi<\pi$ and zero temperature. Referring to Fig. 2, for the right- and left-moving paths, the states with spin parallel to the quantization axis defined in the caption are both populated. As the parameter ϕ varies between $\pm\pi/2$, this quantization axis varies. A net spin is generated along the positive y axis, whereas the x component adds to zero. Analytically, the spin accumulation can be obtained from Eq. (9), which gives

$$S^y = \hbar N_f |\Delta|\left[\cos(\chi/2)\tanh\frac{|\Delta|\sin(\chi/2)}{2T} + \sin(\chi/2)\tanh\frac{|\Delta|\cos(\chi/2)}{2T}\right]. \quad (12)$$

As a function of χ, the spin accumulation S^y for both sides of the interface is plotted in Fig. 4. The present case corresponds to the line with $\mathcal{D}=1$. In addition, S^y is also continuous across the interface for perfect transmission. We note, however, that if the magnitude of the gaps of the two superconductors are unequal, there can also be contributions due to continuum states, as in the case of supercurrent between two unequal-gap s-wave superconductors.[19] Since the Green's function decays as $e^{-2\alpha|x|/v_f|\cos\phi|}$, S^y decays in a distance of the order of coherence length $\hbar v_f/|\Delta|$ away from the interface. The total spin accumulation is of the order $\hbar^2 N_f v_f$ per unit length along the junction.

B. No transmission

In this case all particles are reflected. The behavior of the s-wave superconductor for $x<0$ is trivial, and we shall thus concentrate only on the triplet superconductor on the right. Let us denote the incoming wave vectors by $\underline{\hat{k}}$ and the reflected outgoing wave vectors by $\hat{k}$, with $\hat{k}_x>0$ and $\underline{\hat{k}}_x<0$, see Fig. 1. We label the positions along the quasiparticle path consisting of each pairs of $\hat{k}$ and $\underline{\hat{k}}$ by u, with $u<0$ $(u>0)$ labels the part for $\underline{\hat{k}}(\hat{k})$. $\hat{g}(u)$ is continuous at $u=0$ and can be obtained from Eq. (4) with $\hat{a} \to \hat{a}_p(\hat{k})$ and $\hat{b} \to \hat{b}_p(\underline{\hat{k}})$. Since $\hat{d}(\hat{k}) \neq \hat{d}(\underline{\hat{k}})$, we shall introduce the quantities $C \equiv \hat{d}(\hat{k})\cdot\hat{d}(\underline{\hat{k}})$ and $\vec{D} \equiv \hat{d}(\hat{k}) \times \hat{d}(\underline{\hat{k}})$. Note that $C^2+|\vec{D}|^2=1$. The part of $\hat{g}(0)$ which is diagonal in particle-hole space and even in ϵ_n is found to be

$$\pi\frac{|\Delta_p|^2}{[2\epsilon_n^2+|\Delta_p|^2(1+C)]}\begin{pmatrix}(\vec{D}\cdot\vec{\sigma}) & 0 \\ 0 & \sigma_y(\vec{D}\cdot\vec{\sigma})\sigma_y\end{pmatrix}.$$

For our state, $C=-\cos 2\phi$ and $\vec{D}=\hat{z}\sin 2\phi$. It follows that there are no currents J_j. For a given pair of wave vectors $\hat{k}$ and $\underline{\hat{k}}$, there is in general a finite spin along $\vec{D}\parallel\hat{z}$. However, the contribution from the pairs of wave vectors in opposite directions sum to zero (that is, between the pair with outgoing $\hat{k}$ and incoming wave vector being $-\hat{k}$, or alternatively, $\pm\phi$). Therefore $S^z=0$ and there is no spin accumulation in any direction, which can also be seen by noting that $\hat{g}$ does not contain any $\hat{\sigma}^i$ component. The only finite spin current is J_y^z associated with the $\sigma_z\tau_3$ component and its value at $x=0$ is given by

$$J_y^z = \hbar N_f v_f \int_{-\frac{\pi}{2}<\phi<\frac{\pi}{2}} \frac{d\phi}{\pi}(\sin\phi) T\sum_n \pi \frac{|\Delta_p|^2 D^z}{[2\epsilon_n^2+|\Delta_p|^2(1+C)]}, \tag{13}$$

where D^z is the z component of $\vec{D}$. Since $\hat{g}(\hat{k},\epsilon_n,0)=\hat{g}(\hat{\underline{k}},\epsilon_n,0)$, the angular integral in Eq. (13) has been replaced by twice the contribution due to outgoing wave vectors. The factor $\sin\phi$ is due to $\hat{k}_y=\hat{\underline{k}}_y$. At zero temperature, the spin current density $J_y^z=\hbar N_f v_f \frac{|\Delta_p|}{2}$ at the interface and decays into the bulk within a coherence length. The total spin current is of the order $\hbar^2 N_f v_f^2$.

The physical picture of the spin current is similar to that of the edge current in the so-called chiral superconductors[12] and has been discussed also in, e.g., Ref. 6. Our triplet state consists of ↑↑ pairs and ↓↓ pairs only, with wave functions, respectively, given by $(-d^x+id^y)\rightarrow i(k_x-ik_y)$ and $(d^x+id^y)\rightarrow i(k_x+ik_y)$. Due to the phase difference between the order parameters of the incoming and outgoing momenta, each spin component has a bound state (for a given pair of incident- and reflected-wave wave vectors) but opposite energies, $\epsilon=\mp|\Delta_p|\sin\phi$ for spin up (down), respectively, near the surface. Thus the up (down) spins preferentially occupy the states with positive (negative) y momentum contributing to a net spin current J_y^z. In this picture, it also follows that J_y^x vanishes for $\mathcal{D}=0$.

C. General transmission

In this subsection, we consider a general interface between our singlet and triplet superconductor of (angular and spin independent) transmission coefficient $\mathcal{D}$. We denote the incoming (outgoing) wave vector on the right by $\hat{\underline{k}}$ and $\hat{k}$ and conversely for the left, see Fig. 1. The corresponding Green's functions $\hat{g}(\hat{\underline{k}},x=0_\pm)$ and $\hat{g}(\hat{k},x=0_\pm)$ on the two sides of the spin-inactive interface are related to each other by a set of boundary conditions given in Ref. 20. (See also the Appendix.) It is more convenient to express them in terms of the difference $\hat{g}_d=\hat{g}(\hat{k},x=0_+)-\hat{g}(\hat{\underline{k}},x=0_+)=\hat{g}(\hat{k},x=0_-)-\hat{g}(\hat{\underline{k}},x=0_-)$, which is continuous across the interface and the sums $\hat{s}^{r(l)}=\hat{g}(\hat{k},x=0_{+(-)})+\hat{g}(\hat{\underline{k}},x=0_{+(-)})$. It can be shown that the supercurrent J_x in Eq. (8) and spin currents J_x^i in Eq. (10) across the interface can be expressed solely in terms of the difference $\hat{g}_d$. Note that ϕ, which specifies the angle for $\hat{k}$, is now restricted within $\pm\pi/2$. Moreover, the τ_3 and $\tau_3\hat{\sigma}^i$ components of $\hat{g}_d$ are associated with J_x and J_x^i, respectively. Below, we consider the equal gap case for simplicity in which $\hat{g}_d$ can be worked out analytically via Eq. (A16). The τ_3 component contributing to J_x is given by

$$[\hat{g}_d(\hat{k},\epsilon_n)]_{\tau_3} = \frac{(-\pi)\mathcal{D}^2|\Delta|^4\sin(2\chi)}{4\alpha^2\epsilon_n^2+\mathcal{D}^2|\Delta|^4\sin^2\chi+4(1-\mathcal{D})\alpha^2|\Delta|^2\sin^2\phi}. \tag{14}$$

By numerically performing the sum over the Matsubara frequencies, J_x for arbitrary $\mathcal{D}$ is plotted in Fig. 3. Note that the current is odd and is periodic in the phase difference χ with period π, as noted also in Ref. 15. (See also Sec. II.) Second, we find that none of the $\tau_3\hat{\sigma}^i$ components appear in $\hat{g}_d$ and hence all the spin currents J_x^i across the junction are zero. We note, however, that the (spatial) symmetry argument in Sec. II allows a nonzero J_x^y, as in Eq. (1). Therefore, the vanishing of J_x^y results from other symmetries, which we shall discuss near the end of this section.

At the right side of the interface, the spin accumulation S^i and the spin current J_y^i flowing parallel to the interface can all be expressed in terms of $\hat{s}^r$ solely. Here the $\hat{\sigma}^i$ components in $\hat{s}^r$ are needed for S^i and the $\tau_3\hat{\sigma}^i$ ones are for J_y^i as required in Eqs. (9) and (10). By using Eq. (A17), the $\hat{\sigma}^i$ components are listed below,

$$[\hat{s}^r(\hat{k},\epsilon_n)]_{\hat{\sigma}^i} = 4\pi\mathcal{D}|\Delta|^2 \frac{-i\sin\phi\cos\chi\alpha\epsilon_n\sigma_x+\left[\left(1-\frac{\mathcal{D}}{2}\right)\alpha^2+\frac{\mathcal{D}}{2}\epsilon_n^2\right]\cos\phi\sin\chi\sigma_y\tau_3}{4\alpha^2\epsilon_n^2+\mathcal{D}^2|\Delta|^4\sin^2\chi+4(1-\mathcal{D})\alpha^2|\Delta|^2\sin^2\phi}. \tag{15}$$

The spin accumulation S^x is identically zero because the coefficient in σ_x is odd in ϵ_n and the factor $\sin\phi$ also gives zero after the angular integration. This result is consistent with our symmetry argument in Sec. II. The only finite spin accumulation is S^y which is shown in Fig. 4 due to the $\sigma_y\tau_3$ component in Eq. (15). Note that $S^y(\chi)$ obeys the symmetry in Sec. II and has period 2π. As for the spin current, the only nonvanishing component of $\tau_3\hat{\sigma}^i$ is given by

$$[\hat{s}^r(\hat{k},\epsilon_n)]_{\tau_3\hat{\sigma}^i} = \frac{4\pi|\Delta|^2(1-\mathcal{D})\alpha^2\sin(2\phi)\sigma_z\tau_3}{4\alpha^2\epsilon_n^2+\mathcal{D}^2|\Delta|^4\sin^2\chi+4(1-\mathcal{D})\alpha^2|\Delta|^2\sin^2\phi}. \tag{16}$$

For $\mathcal{D}=0$, J_y^z does not depend on χ. For $\mathcal{D}<1$, the phase dependence comes from the $\sin^2\chi$ term in the denominator. The J_y^z versus the phase difference χ is plotted in Fig. 5 for various $\mathcal{D}$. This spin current is even in χ and is periodic with

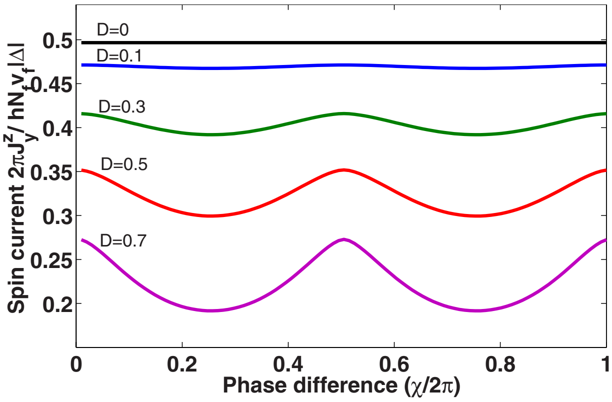

FIG. 5. (Color online) The spin current J_y^z at $x=0_+$. $T=|\Delta|/100$ here. On the singlet side of interface, the spin current is zero for all $\mathcal{D}$.

period π. (see Sec. II). We note that the vanishing of $\tau_3\hat{\sigma}^x$ components leads to zero J_y^x which was not anticipated by our symmetry argument in Sec. II.

At the left side of the interface, S^i and J_y^i can be calculated via $\hat{s}^l$ in Eq. (A18). The $\hat{\sigma}^i$ components are identical to those in $\hat{s}^r$ except that α and ϵ_n are interchanged in the square bracket [...] of Eq. (15) associated with $\sigma_y\tau_3$ component. The numerical results for $S^y(x=0_-)$ are also shown in Fig. 4. It can be seen that S^y is continuous across the interface only for $\mathcal{D}=1$. In addition, all the terms associated with the spin current J_y^i, including the $\sigma_z\tau_3$ component, vanish for all $\mathcal{D}$. Consequently, all the spin currents vanish on the left side.

The vanishing of J_j^i for $x<0$ and J_x^i for all x is a result of spin conservation. Observing that $\sigma_x\tau_3$, σ_y, and $\sigma_z\tau_3$ commute with τ_3 and $\hat{\Delta}_s$, we see that by multiplying Eq. (2) by these matrices and then taking the trace, $\vec{v}_f\cdot\nabla(\mathrm{Tr}[\hat{\sigma}^i\tau_3\hat{g}])=0$, that is, the spin current is constant along any quasiclassical path at any point inside the singlet superconductor. Since the spin current vanishes at $x\to-\infty$, it follows that the spin current on each quasiclassical path vanishes for $x<0$. Hence $J_j^i=0$ for all i and j if $x<0$. Note that this vanishing of the spin current does not rely on angular integration. Since $J_x^i=0$ for $x=0_-$ and the spin current is continuous across a spin-inactive interface ($\hat{g}_d$ is continuous), $J_x^i=0$ also for $x=0_+$. At any point $x>0$, the Green's function is a linear combination of its value at $x=0_+$ and $x\to\infty$ where J_x^i also vanishes. Hence $J_x^i=0$ also for all $x>0$. The vanishing of J_j^i at $x=0_-$ and J_x^i at $x=0$ can also be easily proven using Eqs. (A16) and (A18) using the fact that $\hat{\sigma}^i\tau_3$ commutes with $\hat{g}^l_{\mathrm{aux}}$ and $\mathrm{Tr}[\hat{\sigma}^i\tau_3\hat{g}^l_{\mathrm{aux}}]=0$.

As mentioned, the symmetry allowed J_y^x is found to vanish in our calculation. We have checked that the vanishing of J_y^x is also true in the case of $|\Delta_s|\neq|\Delta_p|$. We do not have a simple physical explanation of this result yet. Mathematically, this follows from the fact that absence of the $\sigma_x\tau_3$ term for $\hat{g}$ on the left of the interface (due to spin conservation) is carried over to $\hat{g}$ on the right. Vorontsov *et al.*[6] also considered the interface between vacuum and a noncentrosymmetric superconductor with finite spin-orbital Rashba energy, which lifts the energy degeneracy between quasiparticles at the same momentum but opposite spin projections. They showed that this can lead to some finite and oscillating J_y^x and J_x^y. We expect that this may also happen in our junction.

IV. DISCUSSIONS AND CONCLUSIONS

We have considered the spin accumulation and spin current near a Josephson junction between a singlet and triplet superconductor. We showed that symmetry arguments (Sec. II) place strong restrictions on the existence of above physical quantities and their dependence on phase difference χ across the Josephson junction. Comparing with the pervious work,[14] this method also applies for any triplet pairing wave function and provides a more general way of determining the direction in which the spin lies. Conversely, the direction and phase dependence of the spin accumulation actually inform us about which symmetry is broken by the junction and hence the symmetry of the triplet order parameter itself. Moreover, the quasiclassical Green's function technique is employed to quantitatively investigate the predicted supercurrent J_x, spin accumulation S^y, and spin current $J_y^{x,z}$. J_y^x turns out to be zero for our junction, though it is symmetry allowed. For transmission coefficient $0<\mathcal{D}<1$ in our calculation, the spin accumulation S^y and spin current J_y^z coexist within a coherence length at the triplet side, a feature which does not appear in the previous studies.[6,14]

In conclusion, we have calculated the spin accumulation and spin current near the interface of a singlet-triplet junction with the triplet order parameter specified by $\hat{d}=k_x\hat{y}-k_y\hat{x}$. The method of quasiclassical Green's functions as well as the symmetry arguments can be generalized to other junction with arbitrary pairing symmetries. These spin accumulation and dissipationless spin currents depend on the phase difference and hence can be controlled by the charge current passing through the junction.

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APPENDIX: EXPLODING AND DECAYING TRICK

In this appendix, we explain the exploding and decaying trick. This trick has been used for pure s-wave[21] and pure p-wave pairing (e.g., Ref. 22). From these references, one can actually deduce that the method can be generalized to mixed singlet and triplet pairs, so that results such as Eq. (4) can still be used. However, we would like to provide our alternate derivation below to show that it is indeed applicable for mixed pairing and, moreover, we believe that our presentation may be more transparent to some readers than those in the literature. We also note that this method is not limited to spatial independent gaps, though we shall discuss only the (piecewise) constant gaps case to simplify our presentation. Furthermore, this method can be easily implemented numerically, as has been performed in, e.g., Refs. 20–22, etc.

We begin by reviewing the first trick for pure s-wave superconductor. Writing u as the parameter along a quasiclassical path, Eq. (2) can be written as

$$[i\epsilon_n\tau_3 - \Delta_s(i\sigma_y)\tau_+ - \Delta_s^*(i\sigma_y)\tau_-,\hat{g}(u)] + iv_f\partial_u\hat{g}(u) = 0, \tag{A1}$$

where we have suppressed the $\hat{k}$ and ϵ_n dependence of $\hat{g}$. A "constant" solution [also satisfying Eq. (3)], which is also the $\hat{g}$ for a bulk superconductor, is given by

$$\hat{g}_{s,\mathrm{bulk}} = -\pi\frac{i\epsilon_n\tau_3 - \Delta_s(i\sigma_y)\tau_+ - \Delta_s^*(i\sigma_y)\tau_-}{(\epsilon_n^2 + |\Delta_s|^2)^{1/2}} \tag{A2}$$

and is thus a linear combination of τ_3, $\sigma_y\tau_+$, and $\sigma_y\tau_-$ matrices only. It is also possible to find other solutions to Eq. (2) [without satisfying Eq. (3)] which are a linear combination of these three matrices only. They are, with $\alpha_s \equiv (\epsilon_n^2 + |\Delta_s|^2)^{1/2}$,

$$\hat{a}_s(u) = e^{-2\alpha_s u/v_f}(-i|\Delta_s|^2\tau_3 - \Delta_s(\alpha_s + \epsilon_n)i\sigma_y\tau_+ + \Delta_s^*(\alpha_s - \epsilon_n)i\sigma_y\tau_-), \tag{A3}$$

$$\hat{b}_s(u) = e^{+2\alpha_s u/v_f}(+i|\Delta_s|^2\tau_3 - \Delta_s(\alpha_s - \epsilon_n)i\sigma_y\tau_+ + \Delta_s^*(\alpha_s + \epsilon_n)i\sigma_y\tau_-), \tag{A4}$$

which will be called the decaying and exploding solutions "in the same block."[22] We note that they satisfy $\hat{a}^2=0$, $\hat{b}^2=0$, and $\{\hat{g}_{s,\mathrm{bulk}},\hat{a}\}=\{\hat{g}_{s,\mathrm{bulk}},\hat{b}\}=0$. In fact, $\hat{g}_{s,\mathrm{bulk}}$ can be written as $\hat{g}=-i\pi(\hat{P}_1-\hat{P}_2)$ with $\hat{P}_1=\hat{a}\hat{b}/\{\hat{a},\hat{b}\}$ and $\hat{P}_2=\hat{b}\hat{a}/\{\hat{a},\hat{b}\}$ being projection operators with $\hat{P}_1+\hat{P}_2=1$, $\hat{P}_1\hat{P}_2=\hat{P}_2\hat{P}_1=0$, and $\hat{a}\hat{P}_1=0$, $\hat{a}\hat{P}_2=\hat{a}$, $\hat{b}\hat{P}_1=\hat{b}$, and $\hat{b}\hat{P}_2=0$ (see, e.g., Ref. 20).

Similar results apply to the pure-triplet superconductor. The bulk solution is

$$\hat{g}_{p,\mathrm{bulk}} = -\pi\frac{i\epsilon_n\tau_3 - \Delta_p(i\vec{d}\cdot\vec{\sigma}\sigma_y)\tau_+ - \Delta_p^*(i\sigma_y\vec{d}\cdot\vec{\sigma})\tau_-}{(\epsilon_n^2 + |\Delta_p|^2)^{1/2}} \tag{A5}$$

and is thus a linear combination of τ_3, $(\vec{d}\cdot\vec{\sigma})\sigma_y\tau_+$, and $\sigma_y(\vec{d}\cdot\vec{\sigma})\tau_-$ matrices only. The other solutions to Eq. (2) [without satisfying Eq. (3)] which are a linear combination of these same three matrices are, with $\alpha_p \equiv (\epsilon_n^2+|\Delta_p|^2)^{1/2}$,

$$\hat{a}_p(u) = e^{-2\alpha_p u/v_f}(-i|\Delta_p|^2\tau_3 - \Delta_p(\alpha_p + \epsilon_n)i(\vec{d}\cdot\vec{\sigma})\sigma_y\tau_+ + \Delta_p^*(\alpha_p - \epsilon_n)i\sigma_y(\vec{d}\cdot\vec{\sigma})\tau_-), \tag{A6}$$

$$\hat{b}_p(u) = e^{+2\alpha_p u/v_f}(+i|\Delta_p|^2\tau_3 - \Delta_p(\alpha_p - \epsilon_n)i(\vec{d}\cdot\vec{\sigma})\sigma_y\tau_+ + \Delta_p^*(\alpha_p + \epsilon_n)i\sigma_y(\vec{d}\cdot\vec{\sigma})\tau_-). \tag{A7}$$

Let us now consider our junction and begin with the case of perfect transmission. $\hat{\Delta}=\Delta_{s,p}$ for $x<(>)0$ and $\hat{g}$ is continuous at $x=0$. Let us first consider $k_x>0$ and label a point on the quasiclassical path as u, with $u=0$ at the interface (hence $u=x/\hat{k}_x$). $\hat{g}$ must decay to $\hat{g}_{s,\mathrm{bulk}}$ ($\hat{g}_{p,\mathrm{bulk}}$) as $u\to-\infty(+\infty)$. We note, however, that we cannot just try the ansatz $\hat{g}(u)=\hat{g}_{p,\mathrm{bulk}}+c_p\hat{a}_p(u)$ for $u>0$ and $\hat{g}(u)=\hat{g}_{s,\mathrm{bulk}}+c_s\hat{a}_s(u)$ for $u<0$ for some scalar coefficients c_p and c_s. This is because the matrices involved for $u>(<)0$ are then different, so $\hat{g}$ being continuous at $u=0$ can never be satisfied. To explain more clearly our idea of solving this problem, let us first consider the special case $\hat{d}=\hat{z}$ so that $i(\vec{d}\cdot\vec{\sigma})\sigma_y=\sigma_x$. Then $\hat{g}_{p,\mathrm{bulk}}$ and $\hat{a}_p$ above are linear combinations of $\tau_3,\sigma_x\tau_\pm$. To find a possible continuous $\hat{g}$ at $u=0$, we must, therefore, include also decaying solutions for $u>0$, which also involves $\sigma_y\tau_\pm$ (due to the singlet superconductor on $x<0$), and an exponentially increasing solution for $u<0$ which also involves $\sigma_x\tau_\pm$. One can find these solutions easily, as done explicitly in Ref. 15. We can, however, also note that these needed solutions can be written as $(\sigma_z\tau_3)\hat{a}_p$ and $(\sigma_z\tau_3)\hat{b}_s$. [Note that $(\sigma_z\tau_3)$ commutes with τ_3, $\hat{\Delta}_s$, and $\hat{\Delta}_p$.] Hence we can try[15]

$$\hat{g}(u) = \hat{g}_{s,\mathrm{bulk}} + c_s\hat{b}_s + \zeta_s(\sigma_z\tau_3)\hat{b}_s \quad u<0, \tag{A8}$$

$$\hat{g}(u) = \hat{g}_{p,\mathrm{bulk}} + c_p\hat{a}_p + \zeta_p(\sigma_z\tau_3)\hat{a}_p \quad u>0, \tag{A9}$$

where $c_{s,p}$ and $\zeta_{s,p}$ are scalar coefficients to be determined. Note that now $\hat{g}$ for both $u<(>)0$ consist of τ_3, σ_3, $\sigma_y\tau_\pm$, and $\sigma_z\tau_\pm$ matrices and hence a solution is possible. Note that Eq. (3) is satisfied. Since $\hat{g}(0)$ can be expressed as either Eq. (A8) and (A9), we can determine the coefficients $c_{s,p}$ and $\zeta_{s,p}$ using simple algebra, but a simpler procedure is to left-multiply Eqs. (A8) and (A9) (at $u=0$) by $\hat{b}_s(0)$ and $\hat{a}_p(0)$, respectively, to obtain

$$\hat{b}_s(0)\hat{g}(0) = \hat{b}_s(0)\hat{g}_{s,\mathrm{bulk}} = -i\pi\hat{b}_s(0), \tag{A10}$$

$$\hat{a}_p(0)\hat{g}(0) = \hat{a}_p(0)\hat{g}_{p,\mathrm{bulk}} = +i\pi\hat{a}_p(0). \tag{A11}$$

Note that the unknown scalar coefficients have all disappeared. Further multiplying Eqs. (A10) and (A11), respectively, by $\hat{a}_p(0)$ and $\hat{b}_s(0)$ and adding the two equations, we thus obtain (hereafter we leave out the argument for simplicity)

$$\hat{g}(0) = -i\pi\{\hat{a}_p,\hat{b}_s\}^{-1}[\hat{a}_p,\hat{b}_s]. \tag{A12}$$

Repeating the above procedure postmultiplication rather than premultiplication actually shows that we can also reverse the order of the anticommutator and commutators in Eq. (A12), as can also be verified explicitly. Note that $\{\hat{a}_p,\hat{b}_s\}$ is a linear combination of $\hat{1}$ and $\hat{\sigma}_z\tau_3$ only.

For $k_x<0$, $u<0(>0)$ corresponds to $x>(<0)$. Following again the above procedure and ensuring that the solutions decay correctly to their respective bulk values at $u\to\mp\infty$ gives us the analogous formula

$$\hat{g}(0) = -i\pi\{\hat{a}_s,\hat{b}_p\}^{-1}[\hat{a}_s,\hat{b}_p]. \tag{A13}$$

Equations (A12) and (A13) are the special examples of Eq. (4) in the present case. For general $\hat{d}(\hat{k})$, to ensure the continuity of $\hat{g}$ at $x=0$, we need matrices τ_3, $\sigma_y\tau_\pm$, $(\vec{d}\cdot\vec{\sigma})\sigma_y\tau_+$, and $\sigma_y(\vec{d}\cdot\vec{\sigma})\tau_-$. A matrix that commutes with τ_3, $\hat{\Delta}_s$, and $\hat{\Delta}_p$ can be seen to be

$$\begin{pmatrix} (\vec{d}\cdot\vec{\sigma}) & 0 \\ 0 & \sigma_y(\vec{d}\cdot\vec{\sigma})\sigma_y \end{pmatrix} \equiv \Sigma_1.$$

The argument above can be repeated with this matrix replacing $\sigma_z\tau_3$ above.

The above argument actually does not depend on the fact that $\sigma_z\tau_3$ (or Σ_1 defined above) be common to both sides of Eqs. (A8) and (A9). To see this, let us first consider the singlet superconductor. We note that the matrices $\hat{1}$, σ_y, $\sigma_x\tau_3$, and $\sigma_z\tau_3$ all commute with τ_3 and $\sigma_y\tau_\pm$, so they are automatically solutions to Eq. (A1). Since the product of two solutions to Eq. (A1) is also a solution, we see that $\hat{g}_{s,\text{bulk}}$, $\sigma_y\hat{g}_{s,\text{bulk}}$, $\sigma_x\tau_3\hat{g}_{s,\text{bulk}}$, and $\sigma_z\tau_3\hat{g}_{s,\text{bulk}}$ are also constant solutions. There are also, in fact, four decaying solutions $\hat{a}_s$, $\sigma_y\hat{a}_s$, $\sigma_x\tau_3\hat{a}_s$, and $\sigma_z\tau_3\hat{a}_s$ and similarly four exponentially increasing solutions. Note that we now have 16 solutions to the 4×4 matrix equation (A1) and, hence, any solution to Eq. (A1) can be written in terms of them. The most general $\hat{g}$ which decays to $\hat{g}_{s,\text{bulk}}$ at $u\to-\infty$ (for $k_x>0$) can be seen to be

$$\hat{g}(u) = \hat{g}_{s,\text{bulk}} + c_s\hat{b}_s + \zeta_{s,1}\sigma_y\hat{b}_s + \zeta_{s,2}\sigma_x\tau_3\hat{b}_s + \zeta_{s,3}\sigma_z\tau_3\hat{b}_s, \tag{A14}$$

where c_s and $\zeta_{s,1-3}$ are scalar coefficients. Note that no constant solution other than $\hat{g}_{s,\text{bulk}}$ can appear on the right-hand side of Eq. (A14) due to the condition at $u\to-\infty$. Since $\hat{1}$, σ_y, $\sigma_x\tau_3$, and $\sigma_z\tau_3$ all commute with τ_3 and $\sigma_y\tau_\pm$, they commute with $\hat{b}_s$. Left multiplication of Eq. (A14) with $\hat{b}_s$ again yields Eq. (A10).

The triplet superconductor on $x>0$ can be treated similarly. For a given $\hat{k}$, we have already noted that the matrix

$$\begin{Bmatrix} [\vec{d}(\hat{k})\cdot\vec{\sigma}] & 0 \\ 0 & \sigma_y[\vec{d}(\hat{k})\cdot\vec{\sigma}]\sigma_y \end{Bmatrix} \equiv \Sigma_1(\hat{k})$$

commutes with τ_3, $[\vec{d}(\hat{k})\cdot\sigma]\sigma_y\tau_+$, and $\sigma_y[\vec{d}(\hat{k})\cdot\sigma]\tau_-$. Two other matrices with this property are (besides $\hat{1}$)

$$\begin{pmatrix} (\vec{d}_{2,3}\cdot\vec{\sigma}) & 0 \\ 0 & -\sigma_y(\vec{d}_{2,3}\cdot\vec{\sigma})\sigma_y \end{pmatrix} \equiv \Sigma_{2,3}(\hat{k}),$$

where $\hat{d}_{2,3}$ are the two vectors orthogonal to $\hat{d}(\hat{k})$. These four matrices $\hat{1}$ and $\Sigma_{1,2,3}(\hat{k})$ are trivial solutions to Eq. (2) for the triplet superconductor. Four other constant solutions are the product between them and $\hat{g}_{p,\text{bulk}}$. Again there are four decaying (increasing) solutions obtained by their product with $\hat{a}_p(\hat{b}_p)$. We again have a total of 16 solutions to Eq. (2) for the triplet superconductor. The most general solution to $\hat{g}(u)$ with $\hat{g}(u)\to\hat{g}_{p,\text{bulk}}$ as $u\to\infty$ (again for $k_x>0$) is

$$\hat{g}(u) = \hat{g}_{p,\text{bulk}} + c_p\hat{a}_p + \zeta_{p,1}\Sigma_1(\hat{k})\hat{a}_p + \zeta_{p,2}\Sigma_2(\hat{k})\hat{a}_p + \zeta_{p,3}\Sigma_3(\hat{k})\hat{a}_p. \tag{A15}$$

On noting that $\Sigma_{1,2,3}(\hat{k})$ commute with τ_3, $[\vec{d}(\hat{k})\cdot\sigma]\sigma_y\tau_+$ and $\sigma_y[\vec{d}(\hat{k})\cdot\sigma]\tau_-$ and hence $\hat{a}_p$, left multiplying Eq. (A15) by $\hat{a}_p$ again yields Eq. (A11). The rest of the demonstration of Eq. (A12) goes through unchanged. Similar argument applies for $k_x<0$.

For finite transmission $\mathcal{D}$, $\hat{g}$ is now, in general, discontinuous at $x=0$ and $\hat{g}$ for incoming, reflected, and transmitted paths are all related. A general boundary condition nonlinear in $\hat{g}$ was first derived independently by Zaitsev[23] and Kieselmann.[24] In Ref. 20, a simplified linearized form of the boundary condition was provided. The derivation given there was for singlet superconductors. However, the boundary conditions derived by Refs. 23 and 24 were actually independent of the assumption on the parities of the superconductors. This can also be checked by using the formulas derived by Millis *et al.*[25] for a spin-active interface between two superconductors of different parities. By ignoring the spin dependence of the scattering amplitudes in Ref. 25 and eliminating the "drone amplitudes" there, one can show that the nonlinear boundary condition of Kieselmann[24] can be recovered. This nonlinear boundary condition can then be linearized using arguments similar to those used in Ref. 20. We express $\hat{g}$ in the form Eqs. (A14) and (A15) for each of the quasiclassical incident, reflected, and transmitted path but with $\hat{g}_{\text{bulk}}$ replaced by $\hat{g}_{\text{aux}}$, the "auxiliary" solution corresponding to the completely reflecting case[20] (that is, for example, $\hat{g}^r_{\text{aux}}$ solves the quasiclassical equation on the quasiclassical path formed by $\underline{\hat{k}}$ and $\hat{k}$ with the physical order parameter on the right but with the boundary condition $\hat{g}^r_{\text{aux}}(\hat{k})=\hat{g}^r_{\text{aux}}(\underline{\hat{k}})$ at $x=0_+$). The decaying and exploding terms can be eliminated using projection operators[20] with arguments similar to those explained above for the perfect transmission case. Thus the derivation in Ref. 20 can be carried over to our present situation. It is most convenient to write the final results in terms of $\hat{s}^{r,l}\equiv\hat{g}(\hat{k},0_\pm)+\hat{g}(\underline{\hat{k}},0_\pm)$ at $x=0_\pm$ and $\hat{g}_d\equiv\hat{g}(\hat{k},0_\pm)-\hat{g}(\underline{\hat{k}},0_\pm)$, where $\hat{k}(\underline{\hat{k}})$ denotes outgoing reflected (incoming incident) wave vector on the right (r). $\hat{k}(\underline{\hat{k}})$ is also the incoming (reflected) wave vector on the left (l). $\hat{g}_d$ (denoted by $\hat{d}$ in Ref. 20) is continuous across the interface and is given by

$$\hat{g}_d = \frac{\frac{i\mathcal{D}}{2\pi}[\hat{g}^r_{\text{aux}},\hat{g}^l_{\text{aux}}]}{1+\frac{\mathcal{D}}{4\pi^2}(\hat{g}^r_{\text{aux}}-\hat{g}^l_{\text{aux}})^2}, \tag{A16}$$

whereas

$$\hat{s}^r = \frac{(2-\mathcal{D})\hat{g}^r_{\text{aux}}+\mathcal{D}\hat{g}^l_{\text{aux}}}{1+\frac{\mathcal{D}}{4\pi^2}(\hat{g}^r_{\text{aux}}-\hat{g}^l_{\text{aux}})^2}, \tag{A17}$$

and

$$\hat{s}^l = \frac{(2-\mathcal{D})\hat{g}^l_{\text{aux}}+\mathcal{D}\hat{g}^r_{\text{aux}}}{1+\frac{\mathcal{D}}{4\pi^2}(\hat{g}^r_{\text{aux}}-\hat{g}^l_{\text{aux}})^2}, \tag{A18}$$

where the subscript "aux" denotes the solution to the $\mathcal{D}=0$ problem. Since $\hat{g}^{r,l}_{\text{aux}}$ commute with the anticommutator $\{\hat{g}^r_{\text{aux}},\hat{g}^l_{\text{aux}}\}$, we need not specify the relative order between the numerator and the denominator in Eqs. (A16)–(A18).

By some straightforward algebra, the complete quasiclassical Green's function $\hat{g}^r_{\text{aux}}$ for $\mathcal{D}=0$ problem in Sec. III B is shown to be

$$\hat{g}^r_{\rm aux} = \frac{(-i\pi)}{\epsilon_n^2 + |\Delta_p|^2 \sin^2\phi}\Big[\alpha_p \epsilon_n \tau_3 + \frac{i}{2}|\Delta_p|^2 \sin(2\phi)\sigma_z\tau_3 + \Delta_p(\alpha_p \sin\phi\sigma_x + \epsilon_n \cos\phi\sigma_y)\sigma_y\tau_+ + \Delta_p^*\sigma_y(\alpha_p\sin\phi\sigma_x - \epsilon_n\cos\phi\sigma_y)\tau_-\Big], \tag{A19}$$

which can be shown to satisfy $(\hat{g}^r_{\rm aux})^2 = -\pi^2$. Together with the trivial $\hat{g}^l_{\rm aux} = \frac{-i\pi}{\alpha_s}(\epsilon_n\tau_3 - \Delta_s\sigma_y\tau_+ - \Delta_s^*\sigma_y\tau_-)$ for the left side, one can obtain $\hat{g}_d$ in the following form:

$$\hat{g}_d = (-i\pi\mathcal{D})\hat{C}\hat{A}, \tag{A20}$$

where the matrix $\hat{C}$ is the inverse of

$$\hat{C}^{-1} \equiv (2-\mathcal{D})\alpha_s[\epsilon_n^2 + (|\Delta_p|\sin\phi)^2] + \frac{\mathcal{D}}{2}\hat{B}. \tag{A21}$$

The matrix $\hat{B}$ comes from the anticommutator and is given by

$$\hat{B} \equiv \alpha_s[\epsilon_n^2 + (|\Delta_p|\sin\phi)^2]\frac{\{\hat{g}^r_{\rm aux},\hat{g}^l_{\rm aux}\}}{(-\pi^2)} = 2\alpha_p\epsilon_n^2 + i\epsilon_n|\Delta_p|^2\sin(2\phi)\sigma_z - 2i\epsilon_n|\Delta_s||\Delta_p|\cos\phi\sin\chi\sigma_y - 2\alpha_p|\Delta_s||\Delta_p|\sin\phi\cos\chi\sigma_x\tau_3 + |\Delta_p|^2\sin(2\phi)(\Delta_s^*\sigma_x\tau_- - \Delta_s\sigma_x\tau_+). \tag{A22}$$

$\hat{A}$ is from the following commutator:

$$\hat{A} \equiv \alpha_s[\epsilon_n^2 + (|\Delta_p|\sin\phi)^2]\frac{[\hat{g}^r_{\rm aux},\hat{g}^l_{\rm aux}]}{(-\pi^2)} = -2\epsilon_n|\Delta_s||\Delta_p|\cos\phi\cos\chi\sigma_y\tau_3 - 2i\alpha_p|\Delta_s||\Delta_p|\sin\phi\sin\chi\sigma_x - 2[\epsilon_n\Delta_p(\alpha_p\sin\phi\sigma_x + \epsilon_n\cos\phi\sigma_y) + \alpha_p\epsilon_n\Delta_s]\sigma_y\tau_+ - 2\sigma_y[\epsilon_n\Delta_p^*(-\alpha_p\sin\phi\sigma_x + \epsilon_n\cos\phi\sigma_y) - \alpha_p\epsilon_n\Delta_s^*]\tau_-. \tag{A23}$$

Similarly, the sum can be expressed as

$$\hat{s}^r = 4\hat{C}\left[\left(1-\frac{\mathcal{D}}{2}\right)\hat{g}^r_{\rm aux} + \frac{\mathcal{D}}{2}\hat{g}^l_{\rm aux}\right]\alpha_s(\epsilon_n^2 + |\Delta_p|^2\sin^2\phi), \tag{A24}$$

and the expression for $\hat{s}^l$ is identical to the one above with interchange of $(1-\frac{\mathcal{D}}{2})$ and $\frac{\mathcal{D}}{2}$ in the bracket.

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